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THE CRITICAL ANGLE OF SEISMIC INCIDENCE AND THE MAXIMUM STRUCTURAL RESPONSE

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SUMMARY

A simple method which can be applied in seismic codes to determine the critical angle of seismic incidence and the corresponding peak response of structures subjected to two horizontal components applied along any arbitrary directions and to the vertical component of earthquake ground motion, is proposed in this paper. The seismic components are given in terms of response spectra that may be equal or have different spectral shapes. The structures are discrete, linear systems with viscous damping. The method, which is based on the response spectrum method of analysis, requires the solution of standard cases of seismic analysis and therefore can be easily implemented in standard computer programs. For the general case of three arbitrary response spectra, the method requires the solution of five seismic loading cases, two for each horizontal component and one for the vertical component. If the horizontal response spectra have the same shape or if there is only one horizontal component, it is then required to solve just two seismic loading cases for the horizontal components and one for the vertical component. It can be shown that the formulas derived for the critical angles and the peak response are essentially identical to the ones obtained earlier by Smeby and Der Kiureghian using random vibration theory.

The application and the accuracy of the method is illustrated by means of numerical analysis of buildings, comparing the results with those obtained using other proposed methods. For the specific case of two horizontal spectra with identical shape and an arbitrary vertical spectra, the critical angle neither depends on the spectral ratio of the two horizontal components nor on the vertical spectrum. For the special case of equal horizontal spectra, the structural response does not vary with the angle of incidence and it is an upper bound for all possible responses.

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KEY WORDS: earthquake direction; critical angle; maximum structural response; seismic components; critical directions; building codes

1. INTRODUCTION

Although in the seismic design of structures the directions of ground motion incidence are usually applied along the fixed structural reference axis, it is known that for most world tectonic regions the ground motion can act along any horizontal direction; therefore, this implies the existence of a possible different direction of seismic incidence that would lead to an increase of structural dynamic response. The maximum structural response associated to the most critical directions of ground seismic motions has been examined in several papers. Wilson and Button[1] proposed a method to calculate the critical angle of incidence and the structural response. Since this method defines the ground motion in terms of response spectra, it can be applied in the practical design of structures. This method is however approximate, as indicated later in this paper, because it does not take into account the proper correlation of ground motion components when they act along the structural principal directions. Smeby and Der Kiureghian[2] using random vibration theory developed an explicit formula to determine the critical angle for the case of two horizontal ground components with

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identical spectral shapes that takes into account the proper correlation between seismic components. For the case of different spectral shapes the authors calculated the spectral moments of the response from which the entire probability distribution can be obtained. Adman Cakiroglu[3] and Cheng and Ger[4] indicated the importance of the seismic directions although the critical angle of incidence was not presented in explicit form. González[5] presented an approximate method to include the effect of seismic direction in the dynamic analysis of buildings, which determines a critical angle in each vibrational mode; the examples presented showed relative errors that differed up to 30 percent of the true values. More recently, Wilson *et al.* discussed the accuracy of the practical procedures used to consider the simultaneous action of two seismic components and proposed a formula to determine the critical earthquake direction; though an improved version of the original one,[1] it still ignored the correlation of ground acceleration components along the structural reference directions.

The main objective of this paper is to develop a simple but accurate method which can be applied in building codes to determine the critical angle and the associated maximum structural response, for the general case of three ground motion components that may have different or identical spectral shape. Preliminary results have been presented previously.[7,8]

2. MAXIMUM STRUCTURAL RESPONSE TO THREE GROUND MOTION COMPONENTS ACTING ALONG TWO ARBITRARY HORIZONTAL AND THE VERTICAL DIRECTIONS

Figure 1 illustrates the situation of a structure subjected to the simultaneous action of two orthogonal horizontal ground accelerations in directions 1 and 2 and a vertical ground acceleration in direction 3 or Z. The component $a_1(t)$ forms an angle θ with the X-axis; X and Y are the reference axis of the structure. Input ground motion is assumed to be represented by a wide-band stationary process. Directions 1,2 and 3 are the principal ground acceleration directions;[9] therefore, ground components a_1, a_2 and a_3 are uncorrelated and are associated to the direction of maximum, intermediate and minimum intensity, respectively, in terms of variances. It is assumed that the major principal axis points in the general direction of the epicentre. Since in most tectonic regions around the world the major principal axis at a site can be directed to several epicentral locations, it seems reasonable as a design criterion to consider all possible values of the angle θ in the range (0°, 360°).

Using random vibration theory the problem stated in Figure 1 was considered in the Reference 2 and the spectral moments of the response were obtained. In this paper, we follow a more simple approach without dealing with random vibrations issues.

Let Sa_1, Sa_2 and Sa_3 be the acceleration response spectra for the ground components a_1, a_2 and a_3, respectively. Assuming linear elastic and discrete systems with viscous damping, the response spectrum method of analysis[10,11] is used to determine the maximum structural response. Let R be the maximum

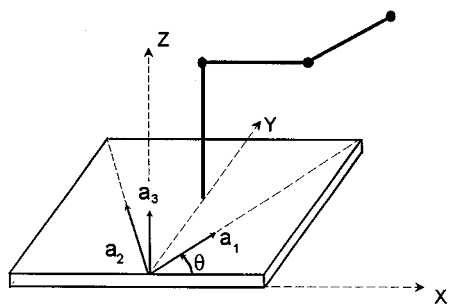


Figure 1. Structure subjected to two horizontal components applied along any arbitrary directions and the vertical component of seismic motion

probable dynamic response or peak response to the simultaneous actions of the three spectra, corresponding to any response parameter such as displacement, stress, force, etc. For spectrum Sa_1, the peak modal response (R_i^1) in the ith mode of vibration, can be written as[7]

$$R_i^1 = R_i^{1x} \cos\theta + R_i^{1y} \sin\theta \tag{1}$$

where R_i^{1x} and R_i^{1y} are the peak modal responses calculated when the spectrum Sa_1 acts along the reference axis of the structure, X and Y, respectively. It should be pointed out that the algebraic sum comes from the fact that ground component ($a_1 \cos\theta$) along direction X and ($a_1 \sin\theta$) along direction Y, are fully correlated.

Similarly, for spectrum Sa_2 the peak modal response (R_i^2) in the vibrational mode ith is given by

$$R_i^2 = R_i^{2y} \cos\theta - R_i^{2x} \sin\theta \tag{2}$$

where R_i^{2x} and R_i^{2y} are the peak modal responses obtained when the spectrum Sa_2 acts along the structural reference directions X and Y, respectively. Finally, let R_i^3 be the peak modal response in mode ith when spectrum Sa_3 acts in the vertical Z direction.

The peak response, R^1, to the seismic component 1 is obtained combining the peak modal responses including modal correlation[12,13]

$$R^1 = \left[\sum_i \sum_j C_{ij} R_i^1 R_j^1 \right]^{1/2} \tag{3}$$

and for seismic components 2 and 3,

$$R^2 = \left[\sum_i \sum_j C_{ij} R_i^2 R_j^2 \right]^{1/2} \tag{4}$$

$$R^3 = \left[\sum_i \sum_j C_{ij} R_i^3 R_j^3 \right]^{1/2} \tag{5}$$

where C_{ij} is the correlation coefficient between responses in modes i and j.

Since ground components are uncorrelated, the peak response, R, to the simultaneous action of the three components is given by

$$R = [(R^1)^2 + (R^2)^2 + (R^3)^2]^{1/2} \tag{6}$$

From Equations (1)–(6), the peak response R is obtained as a function of the angle of incidence (θ):

$$R(\theta) = \Bigg\{ [(R^{1x})^2 + (R^{2y})^2]\cos^2\theta + [(R^{1y})^2 + (R^{2x})^2]\sin^2\theta + 2\sin\theta\cos\theta \left[\sum_i \sum_j C_{ij} R_i^{1x} R_j^{1y} - \sum_i \sum_j C_{ij} R_i^{2y} R_j^{2x} \right] + (R^3)^2 \Bigg\}^{1/2} \tag{7}$$

where

$$R^{1x} = \sum_i \sum_j C_{ij} R_i^{1x} R_j^{1x} \tag{8}$$

$$R^{1y} = \sum_i \sum_j C_{ij} R_i^{1y} R_j^{1y} \tag{9}$$

$$R^{2x} = \sum_i \sum_j C_{ij} R_i^{2x} R_j^{2x} \tag{10}$$

$$R^{2y} = \sum_i \sum_j C_{ij} R_i^{2y} R_j^{2y} \tag{11}$$

The function $R(\theta)$ describes the variation of the peak response with the incidence angle, θ, of the principal horizontal ground motion components. $R(\theta)$ is periodical, with a period equal to 180°. For $\theta = 0°$, the three ground spectra are applied along the structural reference directions (X, Y, Z) and the maximum response is given by the well-known formula (SRSS)

$$R = \{(R^{1x})^2 + (R^{2y})^2 + (R^3)^2\}^{1/2} \tag{12}$$

When $\theta \neq 0$, Equation (7) includes the correlation of the seismic components 1 and 2 along the structural reference directions.

It is convenient to point out that Equation (7) expresses the peak response for any arbitrary angle of incidence, in terms of the peak responses associated with the solution of five basic cases of seismic loading where the spectra are applied along the structural reference directions. These cases are defined as follows:

(i) Apply spectrum 1 along the direction X and analyse the structure to calculate the modal responses R_i^{1x} and the peak dynamic response which will be called R^{1x} (Equation (8)).
(ii) Similarly, apply spectrum 1 along the direction Y and calculate R_i^{1y} and R^{1y} (Equation (9)).
(iii) Similarly, apply spectrum 2 along the X direction and obtain R_i^{2x} and R^{2x} (Equation (10)).
(iv) Similarly, apply spectrum 2 along the Y direction and obtain R_i^{2y} and R^{2y}, (Equation (11)).
(v) Finally, apply spectrum 3 along the Z direction to obtain R_i^3 and R^3 (Equation (5)).

2.1. *Critical angle*

The critical angle, θ_c, is defined by the value of θ that renders the maximum value of R in equation (7). Taking the derivative of R with respect to θ and setting it equal to zero, we get

$$\theta_c = \frac{1}{2}\tan^{-1}\left\{\frac{2\sum_i\sum_j C_{ij}\ [R_i^{2y} R_j^{2x} - R_i^{1x} R_j^{1y}]}{(R^{1y})^2 + \ (R^{2x})^2 - (R^{1x})^2 - (R^{2y})^2}\right\} \tag{13}$$

Therefore, equation (13) gives two roots for θ_c, separated 90° between each other, that define the maximum and the minimum values of the peak structural response R in Equation (7). It should be noted that the critical angles depend on the characteristics of the horizontal spectra although they do not depend on the structural response to the vertical component (R^3) and hence do not depend on the vertical spectrum. Furthermore, as expected, the critical angles depend on the response parameter being considered.

2.2. *Proposed method of analysis to determine critical response*

If the general direction of the epicentre is known, Equation (7) can be used directly to determine the critical response. If not, it is necessary to calculate the critical angle of ground motion incidence. The results presented above indicate that it is not necessary to make a separate seismic analysis for each possible angle of incidence in order to pick the critical response; instead, only five seismic analysis are required in combining results according to the equations shown above.

Next we present a summary of the steps proposed in this paper in order to determine the critical angle of incidence and the corresponding maximum peak response to an arbitrary vertical spectrum and to two arbitrary horizontal spectra applied along any arbitrary directions 1 and 2 that form a given angle θ with the structural reference directions X and Y (Figure 1):

(1) Solve the five basic cases of seismic loading defined previously, to obtain the modal responses R_i^{1x}, R_i^{1y}, R_i^{2x}, R_i^{2y} and R_i^3, and the peak responses R^{1x}, R^{1y}, R^{2x}, R^{2y} and R^3 given by Equations (8)–(11) and (5).
(2) Determine the two critical values of the angle θ from Equation (13).

(3) For each of the two angles obtained, determine the peak response R from Equation (7). The highest value is the maximum peak response for all possible angles of incidence. It should be mentioned that each response value may have a different critical angle and hence the method should be applied to each of the desired response values.

2.3. *An example of application*

Only horizontal ground motions are considered because the critical directions do not depend on the vertical motion. The one-storey reinforced concrete building shown schematically in Figure 2 is asymmetrical in both directions with eccentricities $e_x = 0{\cdot}33B_x$ and $e_y = 0{\cdot}38B_y$, where $B_x = 6$ m and $B_y = 8$ m. The rigidity centre is at the geometric centre of the floor plant. Beam and column sections are 40×60 and 60×60 cm^2, respectively. Storey weight is 65 t. Damping ratio is 5 percent for all modes. Storey height is 4 m. The floor is modelled as a rigid horizontal diaphragm with infinite in-plane stiffness and no out-of-plane stiffness, with three dynamic degrees of freedom (displacements x and y and rotation about the vertical axis). The structure is modelled as a space frame considering flexural, axial, shear and torsional deformations. Natural periods for the three modes are 0·20, 0·16, and 0·065 sec.

The structural response, R, is determined in terms of the torsional moment, T, referred to the centre of mass (Figure 2). The building is subjected to the two horizontal response spectra of El Centro 1940 earthquake records[17] shown in Figure 3. The SOOE component is applied in direction 1 and the S90W component in direction 2 (Figure 2).

In order to determine both critical angles and the maximum structural response, the building was analysed according to the proposed method, as described in Section 2.2. Since there is no vertical ground motion, $R^3 = 0$ and there are only four cases of seismic analysis. The correlation coefficients C_{ij} required in equations (13) and (7) are taken from Reference 15.

The four cases of seismic loading defined previously were solved using the computer program SAP90.[14] The peak modal responses for each basic case are indicated in Table I. The correlation coefficients are $C_{12} = 0{\cdot}16$, $C_{13} = 0{\cdot}01$ and $C_{23} = 0{\cdot}01$. The peak responses (equations (8)–(11)) when spectrums 1 and 2 are applied along the building reference directions X and Y, are

$$R^{1x} = 284{\cdot}2 \text{ t m}, \qquad R^{2x} = 223{\cdot}2 \text{ t m}$$

$$R^{1y} = 221{\cdot}6 \text{ t m}, \qquad R^{2y} = 174{\cdot}2 \text{ t m}$$

From equation (13), the critical angles are: $\theta_{c1} = 52{\cdot}11^\circ$ and $\theta_{c2} = 142{\cdot}11^\circ$. The critical responses calculated from Equation (7) are 360·25 t m for the maximum moment and 283·3 t m for the minimum. Finally, the torsional moment given by equation (7) is shown in Figure 4 plotted against the angle of incidence.

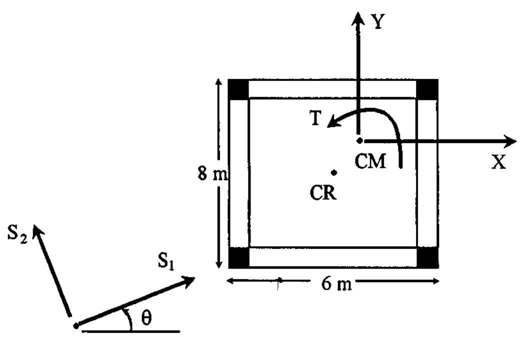


Figure 2. One-storey building: CR = centre of rigidity; CM = centre of mass

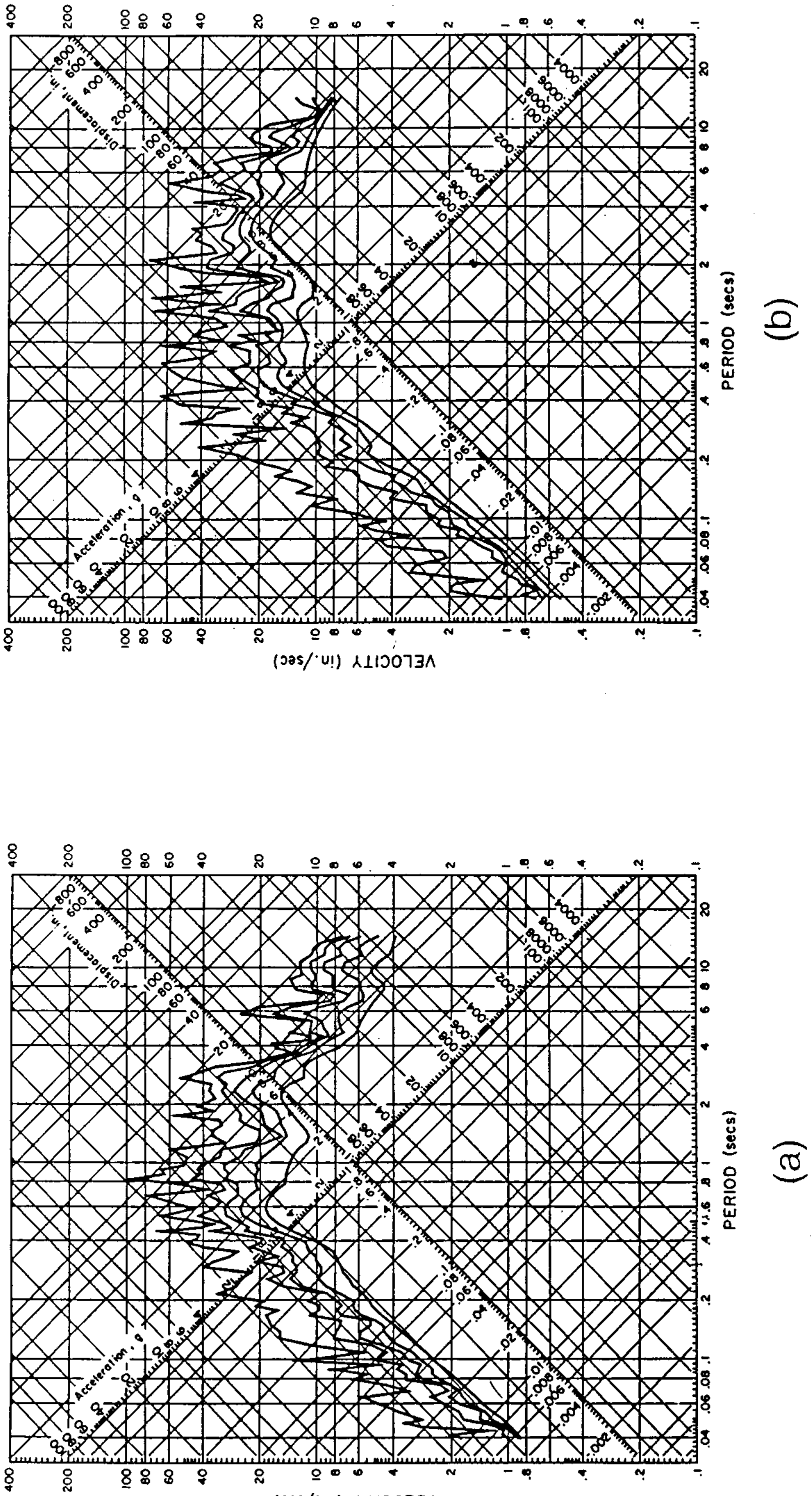


Figure 3. Response spectra for El Centro records, Imperial Valley Earthquake, 18 May 1940, for damping values of 0, 2, 5, 10 and 20% (Reference 17): (a) SOOE component; (b) S9OW component

Table I. Peak modal responses for the torsional moment (t m)

Mode	R_i^{1x}	R_i^{2x}	R_i^{1y}	R_i^{2y}
1	− 282·5	− 222·0	223·2	175·4
2	− 9·35	− 6·92	− 12·1	− 8·93
3	− 1·62	0·83	− 0·96	− 0·49

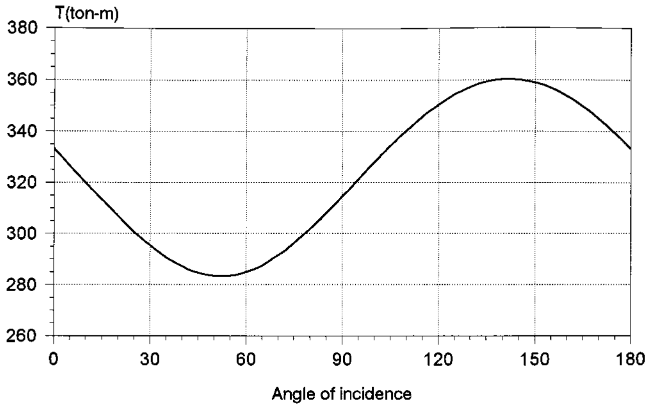


Figure 4. Torsional moment, T, for any angle of incidence. One-storey building subjected to the two horizontal components of El Centro, 1940

3. THE PRACTICAL CASE OF HORIZONTAL GROUND MOTION COMPONENTS WITH IDENTICAL SPECTRAL SHAPE AND AN ARBITRARY VERTICAL SPECTRA

Let us assume the following relationship between the horizontal spectra: $Sa_2 = \alpha\, Sa_1$, where α is defined as the spectral ratio for the horizontal components of ground motion. This assumption is usually adopted in the earthquake design of structures. Since direction 1 is the direction of maximum intensity, α is a number between 0 and 1. The vertical spectra (Sa_3) may have any arbitrary shape.

The peak responses defined before are simplified as follows: $R^{2x} = \alpha R^{1x}$, $R^{2y} = \alpha R^{1y}$, as well as the peak modal responses $R_i^{2y} = \alpha R_i^{1y}$ and $R_i^{2x} = \alpha R_i^{1x}$. Substituting these expressions into equation (7), the peak response R is given as a function of the angle of incidence:

$$R = \left\{ [(R^{1x})^2 + (\alpha R^{1y})^2] \cos^2\theta + [(\alpha\, R^{1x})^2 + (R^{1y})^2] \sin^2\theta \right.$$
$$\left. + 2 \sin\theta \cos\theta\, (1 - \alpha^2) \sum_i \sum_j C_{ij}\, R_i^{1x}\, R_j^{1y} + (R^3)^2 \right\}^{1/2} \quad (14)$$

where it is convenient to indicate that the peak response R for any angle of incidence is given in terms of the peak responses calculated from the solution of three cases of seismic loading which are defined as follows:

(i) Apply spectrum 1 along the X direction. Calculate the modal response R_i^{1x} and the peak response R^{1x}.
(ii) Apply spectrum 1 along the Y direction. Calculate the modal response R_i^{1y} and the peak response R^{1y}.
(iii) Apply spectrum 3 along the Z direction. Calculate the modal response R_i^3 and the peak response R^3.

Taking the derivative of Equation (14) with respect to θ and setting it equal to zero, we get the following expression for the critical angles:

$$\tan 2\theta_c = \left\{\frac{2\sum_i \sum_j C_{ij} R_i^{1x} R_j^{1y}}{(R^{1x})^2 - (R^{1y})^2}\right\} \frac{(1-\alpha^2)}{(1-\alpha^2)} \tag{15}$$

where the last factors of equation (15) are retained to remark that the critical angles are undefined for $\alpha = 1$. Therefore, for $\alpha \neq 1$, the critical angles are

$$\theta_c = \frac{1}{2} \tan^{-1} \left\{\frac{2\sum_i \sum_j C_{ij} R_i^{1x} R_j^{1y}}{(R^{1x})^2 - (R^{1y})^2}\right\} \tag{16}$$

Equation (16) gives two critical angles, with a separation of 90°, which when substituted into equation (14) lead to the maximum and the minimum peak structural responses. It should be pointed out that the critical angles do not depend on α. This means that the critical angles are the same whether we have one or two seismic horizontal components. Also, as found for the case of arbitrary spectra, the critical angles do not depend on the response to the vertical ground motion.

It is possible to demonstrate that Equations (14) and (16) are identical to the ones found by Smeby and Der Kiureghian[2] from the use of concepts of random vibrations. Equations (14) and (16) differ from those presented by Wilson and Button[1] and Wilson *et al.*[6] because we have incorporated the effect that in each mode has the simultaneous action of two seismic components that are totally correlated. The formulas of the Named references 1 and 6 can be obtained from Equation (14) if we consider just the effect of one mode of vibration and hence they can be interpreted as an approximation to the true critical response.

3.1. *Proposed method of analysis*

A summary of the steps to determine the critical angles of incidence and the corresponding maximum peak structural response when horizontal spectra along directions 1 and 2 have identical shapes, consists of the following

(1) Solving the three cases of seismic loading defined previously to obtain the modal responses R_i^{1x}, R_i^{1y} and R_i^3, and the peak responses R^{1x}, R^{1y} and R^3 given by Equations (8), (9) and (5).
(2) Determining the two critical angles from Equations (16).
(3) For the two critical angles, determining their critical responses from Equation (14); the highest response is the maximum peak response for all possible angles of incidence.

3.2. *The special case of equal spectra applied in the horizontal directions 1 and 2*

The peak response for the particular case in which the same spectrum acts in the two horizontal directions 1 and 2, is obtained using $\alpha = 1$ in equation (14):

$$R = \{(R^{1x})^2 + (R^{1y})^2\}^{1/2} \tag{17}$$

from which we can observe that the peak response does not depend on the value of the angle of incidence; any value of θ is a critical angle as also indicated by Equation (15). For this reason, it is enough to analyse the typical case of $\theta = 0°$ to determine the maximum peak response to any angle of incidence. This unexpected outcome has application in seismic design because the value of R given by equation (17) is an upper bound of all possible responses due to any combination of spectral ratios and angles of incidence. If a more detailed

analysis is required to avoid being so conservative, then a value of the spectral ratio α less than one should be adopted and Equations (16) and (14) should be used to obtain the critical angle and the maximum peak response as indicated before.

3.3. *Example of application and comparison of results*

Since the critical angles do not depend on the structural response to vertical ground motion, only horizontal ground motions are considered in this example.

A nine-storey reinforced concrete asymmetric building is indicated in Figure 5. The building dimensions have been selected according to the Venezuelan Building Code which has similar requirements with California's standards. Dimensions of selected frames and walls along directions X and Y are also indicated in the figure. Storey weights are 775 t for the first storey, 400 t for the top storey and 590 t for the rest. The centre of mass is at the geometric centre of the floor. Each floor is modelled as a diaphragm with infinite in-plane stiffness (3 degrees of freedom per floor). The structure is modelled as a space frame including flexural, axial, shear and torsional deformations. Damping ratio is 5 percent for all modes. The periods and effective mass in directions X and Y are indicated in Table II. Additional information and detailed results are given by Torres.[16]

The building is subjected to two spectra applied along arbitrary horizontal directions as indicated in Figure 5(a). The acceleration spectrum (Sa_1) of the major component in direction 1 is shown in Figure 6; the minor component (Sa_2) in direction 2 is defined by four different spectral ratios: $\alpha = 0$, 0·5, 0·85 and 1. The value $\alpha = 0$ describes the application of a single horizontal spectrum along direction 1, whereas $\alpha = 1$ implies the application of two horizontal spectrum of equal intensity in directions 1 and 2. The value 0·85 is representative of several recorded ground motions.

The peak structural response, R, is described by two parameters: The base shear in the Y direction, V_y, and the base torsional moment, T, referred to the centre of mass.

The building was analysed according to the following two procedures.

(i) The proposed method, as described in Section 3.1 and defined by equations (14 and 16). The correlation coefficients C_{ij} are taken from Reference 15. Since there is no vertical ground motion, only two basic cases of dynamic analysis are required, with spectrum Sa_1 applied first along the X and then along the Y direction; Both analysis were done with SAP90· The modal responses, R_i^{1x} and R_i^{1y}, for 12 vibrations modes are presented in Tables III and IV for the base shear V_y and the base torsional moment T, respectively. The peak responses for the two basic cases are:

$$\text{For } V_y\text{:}\quad R^{1x} = 361{\cdot}0\ \text{t}, \qquad R^{1y} = 1697{\cdot}6\ \text{t}$$

$$\text{For } T\text{:}\quad R^{1x} = 17\,794\ \text{t m}, \qquad R^{1y} = 26\,389\ \text{t m}$$

(ii) The WSH procedure, as presented in Reference 6, was used to calculate the critical angles.

The base forces, V_y and T, obtained with equation (14) are plotted in Figure 7 against the angle of incidence, θ, for the four values of the spectral ratio α.

Critical angles determined from the Proposed Method [Equation (16)] and the WSH procedure are shown in Table V; as expected, the critical angles calculated with the WSH procedure are not correct for the reasons given in Section 3.

From Figure 7 it is convenient to emphasize that when both horizontal components have the same spectra ($\alpha = 1$), the peak response does not vary with the angle of incidence. This response value is an upper bound for all possible responses. Also, the critical angles are independent of the spectral ratio α, which was apparently first pointed out by Smeby and Der Kiureghian.[2]

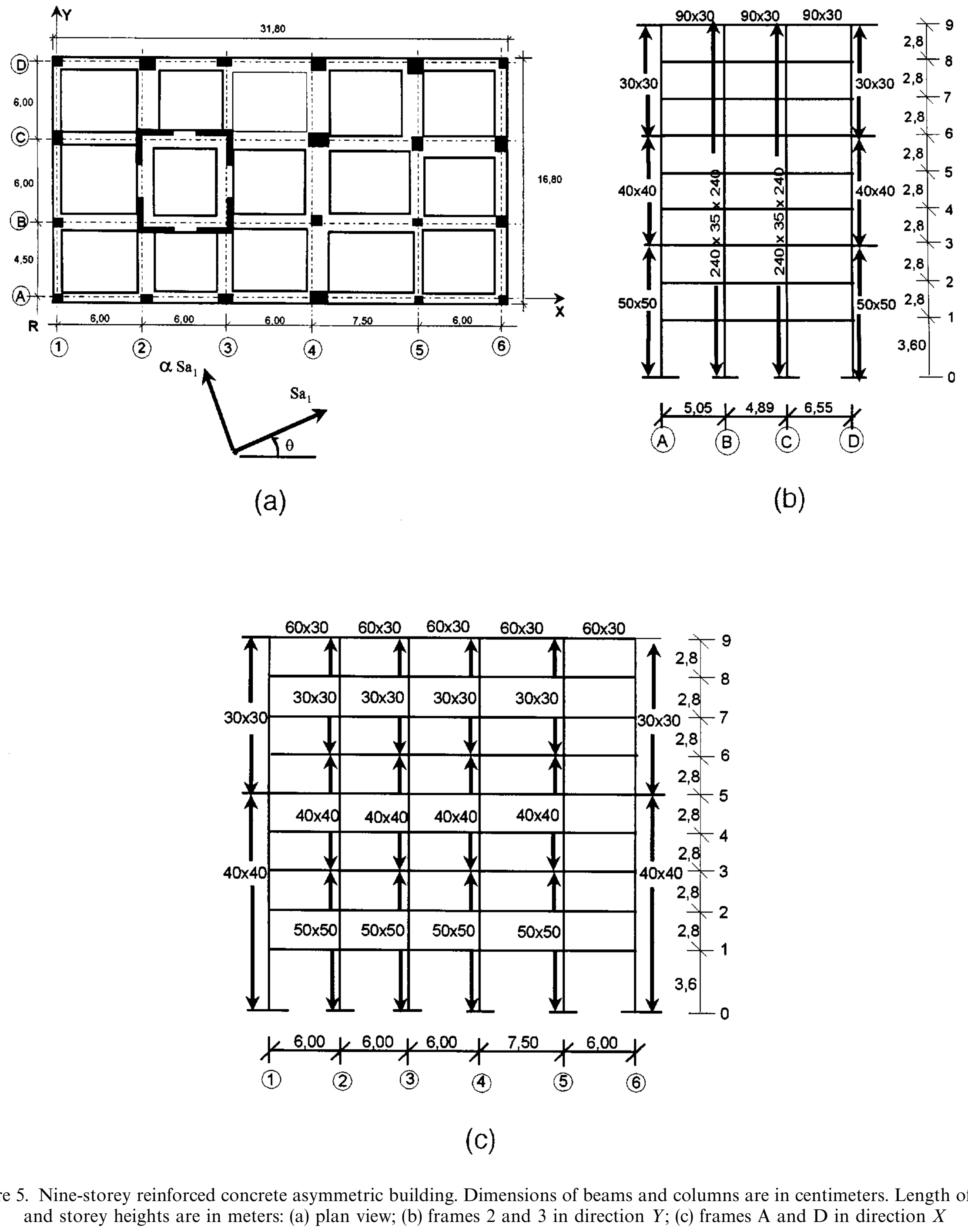


Figure 5. Nine-storey reinforced concrete asymmetric building. Dimensions of beams and columns are in centimeters. Length of bays and storey heights are in meters: (a) plan view; (b) frames 2 and 3 in direction Y; (c) frames A and D in direction X

Table II. Periods and effective mass for the first 12 modes of the 9-storey building

Mode	Period (sec)	E_{mx} (%)	E_{my} (%)
1	1·034364	0·410	32·756
2	0·549200	73·990	1·228
3	0·464436	0·815	38·996
4	0·344391	0·110	8·327
5	0·198641	0·020	1·204
6	0·165850	16·574	0·258
7	0·138879	0·181	11·312
8	0·135074	0·001	0·001
9	0·102521	0·008	0·488
10	0·086154	4·713	0·070
11	0·081793	0·000	0·138
12	0·071995	0·047	3·069

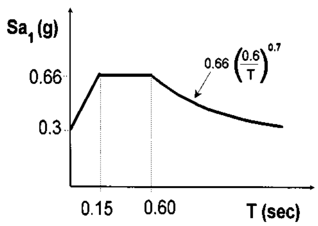


Figure 6. Response spectrum for the major horizontal component

Table III. Modal responses for base shear force V_y (t)

Mode	R_i^{1x}	R_i^{1y}
1	− 87·772	784·910
2	333·710	42·993
3	− 197·420	1365·400
4	− 33·561	291·560
5	− 5·450	42·158
6	72·380	9·028
7	− 48·093	380·070
8	− 0·024	0·027
9	− 1·760	14·138
10	15·457	1·886
11	− 0·209	3·645
12	− 9·559	76·984

Table IV. Modal responses for base torsional moment T (t m)

Mode	R_i^{1x}	R_i^{1y}
1	– 2554·700	22845·000
2	– 16580·000	– 2136·000
3	– 1406·800	9729·800
4	– 791·370	6875.100
5	– 185·070	1431·400
6	– 3690·100	– 460·270
7	– 507·260	4008·800
8	14·681	– 16·825
9	– 49·353	396·260
10	– 807.730	– 98·549
11	– 6·957	121·080
12	– 88·517	712·870

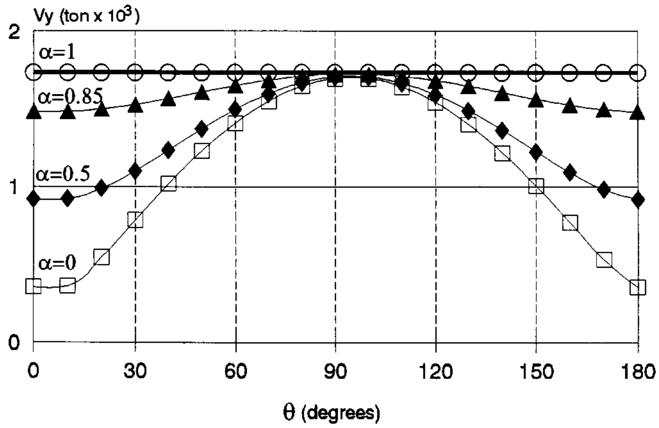
4. CONCLUSIONS

(i) The proposed method to calculate the critical angles of ground motion incidence and the critical structural response to two arbitrary horizontal and a vertical spectra, requires only the solution of five basic cases of seismic loading, defined when the spectra are applied along the three structural reference directions: Two analysis corresponding to each horizontal spectra and one to the vertical spectrum. If there is only one horizontal seismic component or if the two horizontal spectra have identical shape, only three loading cases are required, two with the horizontal spectrum and one with the vertical spectrum. The basic difference with the work presented in Reference 2 for identical horizontal spectral shape, is that the formulas for the critical angles and the peak responses have been derived using a more simple approach based on accepted techniques of earthquake analysis: CQC rule for combining modal responses and SRSS rule for combining responses to principal ground motion components.

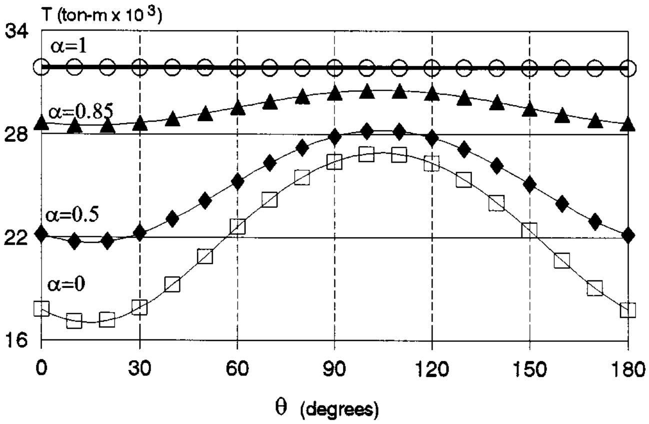
(ii) The method is accurate and its application has been illustrated by means of dynamic analysis of asymmetric buildings. The method can be easily implemented in the existing computer programs for practical applications and included in seismic codes for the design of irregular structures where these effects are significant.

(iii) The critical direction of the horizontal ground motion components defined as the direction that yield the maximum structural response, depends on the two horizontal spectra and also depends on the structural response parameter analysed; it does not depend on the vertical spectrum. For the practical case of two horizontal spectra with identical shapes and an arbitrary vertical spectrum, the critical angle neither depends on the spectral ratio of the two horizontal components nor on the vertical spectrum; in this case the critical angle is the same whether one or two seismic horizontal components are applied.

(iv) For the special case of identical spectra along the two horizontal directions, the structural response does not vary with the angle of incidence, i.e. any direction is a critical direction. This response value is an upper bound to all possible structural responses due to any combination of spectral ratios and angles of incidence. This outcome has application in seismic design; a conservative design criteria would be to analyse the structure with the same horizontal spectrum applied simultaneously along the two horizontal structural reference directions, and the vertical spectrum.



(a)



(b)

Figure 7. Base forces for any angle of incidence and for four values of the spectral ratio α for the nine-storey asymmetric building: (a) base shear in direction Y; (b) base torsional moment

Table V. Critical angles for base shear and base moment

		V_y		T	
		θ_{c1}	θ_{c2}	θ_{c1}	θ_{c2}
Proposed Method		4·7°	94·7°	14·4°	104·4°
WSH procedure*	+	78·0°	168.0°	56·0°	146·0°
	−	12·0°	102·0°	34·0°	124·0°

* Calculated using both signs ($\pm$) in equation (8)

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